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Hydromagnetic waves in an ideally conducting medium with a two-dimensional inhomogeneous magnetic field.

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Possibility of the propagation of hydromagnetic waves is explored in an ideally conducting incompressible fluid mass immersed in a two-dimensional inhomogeneous magnetic field. The magnetic field lies in a plane perpendicular to the surface of the fluid and varies along the normal to the above plane. Because of the absence of the boundary conditions, it is not possible to get a dispersion relation. If the components of the magnetic field are linearly related and its pressure decays exponentially, it is found that the hydromagnetic waves propagate for all values of the wave number and frequency.

1. Introduction

Gajewski & Winterberg (1963), have discussed the propagation of the small amplitude hydromagnetic waves in an ideally conducting uniform medium embedded in an inhomogeneous magnetic field of constant direction. Uberoi (1964), has reconsidered the problem by taking into account the perturbation in pressure and established that the medium is non-dispersive. We have considered the case of a two-dimensional inhomogeneous magnetic field lying in a plane perpendicular to the surface of the fluid and varying in a direction normal to its own plane. It is again found that the medium is non-dispersive and waves can propagate for all wave numbers and frequencies in a direction orthogonal to the direction of inhomogeneity of the magnetic field.

Linearised Equations

Consider, in the state of equilibrium, an ideally conducting incompressible fluid mass of uniform density ρ and pressure p immersed in a time-independent magnetic field $\vec{H}$. We choose a Cartesian coordinate system and its coordinate axes are oriented in such a way that its xy-plane coincides with the surface of the fluid mass and yz-plane contains the inhomogeneous magnetic field $\vec{H}$. The magnetic field varies as a function of x alone and is independent of y, z and t.

Taking perturbations in velocity, magnetic field, pressure and gravitational potential as $\vec{v}$, $\vec{h}$, δp and δU respectively, the linearised equations are

(a) Equation of continuity : $\operatorname{div} \vec{v} = 0,$ (1)

(b) Equation of motion : $\rho \frac{\delta \vec{v}}{\sigma t} = - \nabla \delta p - \rho \nabla \delta U$

$$+ \frac{\mu}{4\pi} \{ (\operatorname{curl} \vec{H}) \times \vec{h} + (\operatorname{curl} \vec{h}) \times \vec{H} \} , \tag{2}$$

and

Maxwell's equations : $\frac{\delta \vec{h}}{\delta t} = \operatorname{curl} (\vec{v} \, x \, \vec{H}),$ (3)

$$\operatorname{div} \vec{h} = 0. \tag{4}$$

Component Equations

Choosing the dependence of the perturbed quantities on y, z and t as

$$f (x) \exp i (\omega t + k_1 y + k_2 z) , \tag{5}$$

equation (1) becomes (omitting the exponenential factor)

$$D v_x = - i (k_1 v_y + k_2 v_z) . \tag{6}$$

In view of relation (5) and using equation (6), the components of equation (2) and equation (3) can be written as

$$\rho \omega v_x = \mathrm{iD} (\delta p + \rho \delta U) \tfrac{\mu}{4\pi} \{ L h_x + \mathrm{it} (H_y h_y + H_z h_z) \} , \tag{7}$$

$$\rho \omega v_y = k_1 (\delta p + \rho \, \delta U) - \tfrac{\mu}{4\pi} \{ i (D H_y) h_x - k_2 H_z h_y - (M - k_1 H_z) h_z \}, \tag{8}$$

$$\rho \omega v_z = - k_2 (\delta p + \rho \delta U) - \tfrac{\mu}{4\pi} \{ i (D H_z) h_x + (M + k_2 H_y) h_y - k_1 H_y h_z \}, \tag{9}$$

$$\vec{h} = [L v_x , i (D H_y) v_x + L v_y , i (D H_z) v_x + L v_z] ; \tag{10}$$

where

$$L = k_1 H_y + k_2 H_z , \; M = k_2 H_y - k_1 H_z . \tag{11}$$

Eliminating $\vec{h}$ from equations (7)–(9) with the help of equation (10). we get

$$2(\omega^2-\alpha)\{(\omega^2-\alpha)^2+\alpha\beta\}D^2v_x-\{(2\alpha'-\beta')(\omega^2-\alpha)^2-(\alpha'\beta+2\alpha\beta')(\omega^2-\alpha)-2\omega^2\alpha'\beta\}Dv_x-\{2(k_1^2+k_2^2)(\omega^2-\alpha)^3-\beta''(\omega^2-\alpha)^2-(\alpha'\beta)(\omega^2-\alpha)-(\alpha')^2\beta\}v_x=0, \tag{12}$$

$$v_y=\frac{2i\{k_1(\omega^2-\alpha)+k_2(\alpha\beta)^{1/2}\}Dv_x+ik_2\alpha'(\beta/\alpha)^{1/2}v_x}{2(k_1^2+k_2^2)(\omega^2-\alpha)}, \tag{13}$$

$$v_z=\frac{2i\{k_2(\omega^2-\alpha)-k_1(\alpha\beta)^{1/2}\}Dv_x-ik_1\alpha'(\beta/\alpha)^{1/2}v_x}{2(k_1^2+k_2^2)(\omega^2-\alpha)}; \tag{14}$$

where

$$\alpha=(\mu/4\pi\rho)L^2\ ,\ \beta=(\mu/4\pi\rho)M^2\ , \tag{15}$$

and the quantities L and M are given by eqation (11). The prime and D stand for differentiation with respect to x.

Discussion

Since there are no boundary conditions to be satisfied it is not possible to get a dispersion relation. But the amplitude of the waves if propagate may be determined from equations (12) to (14).

If the components of the magnetic field are linearly related by

$$H_y=(k_1/k_2)H_z\ , \tag{16}$$

then in view of assumption (16), equation (15) becomes

$$\alpha=(Q/k_2^2)(k_1^2+k_2^2)^2\ ,\ \beta=0\ , \tag{17}$$

where

$$Q=(\mu/4\pi\rho)H_3^2. \tag{18}$$

With the help of equation (17), equations (12) – (14) are reduced to

$$D^2v_x-\frac{(k_1^2+k_2^2)^2Q'}{k_2^2\omega^2-(k_1^2+k_2^2)^2Q}\ Dv_x-(k_1^2+k_2^2)v_x=0\ , \tag{19}$$

$$v_y=[ik_1/(k_1^2+k_2^2)]Dv_x\ , \tag{20}$$

$$v_z=[ik_2/(k_1^2+k_2^2)]Dv_x. \tag{21}$$

From equation (18) it follows that Q is proportional to the magnetic pressure ($\mu H^2 8\pi$). Assuming the dependence of the magnetic pressure on x, of the form

$$Q=[k^2{}_2\omega^2/(k_1{}^2+k_2{}^2)^2]+Ke^{-A|x|}, \tag{22}$$

where K and A are arbitrary constants, equation (19) takes the form

$$D^2v_x-A\,Dv_x-(k_1^2+k_2^2)v_x=0,$$

and it admits a solution of the from

$$v_x=B\exp[A-\{A^2+4(k_1^2+k_2^2)\}^{1/2}]\,|\,x\,|\,/_2, \tag{23}$$

where B is an arbitrary constant and other constant becomes zero as $v_x \to 0$ for large values of x.

Equation (23), (20) and (21) establish that the hydromagnetic waves can propagate in a direction orthogonal to the direction of the inhomogeneity of the magnetic field for all values of ω, k_1 and k_2 under the assumptions (16) and (22).

REFERENCES

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